Lab 16: Chapters 12

1. A random sample is selected from a population with mean $\mu = 60$ and standard deviation $\sigma = 3$. Determine the mean and standard deviation of the $\bar{x}$ sampling distribution for each of the following sample sizes
   (a) $n = 6$
   (b) $n = 18$
   (c) $n = 42$
   (d) $n = 75$
   (e) $n = 200$
   (f) $n = 400$

2. For which of the sample sizes given in the previous exercise would it be reasonable to think that the $\bar{x}$ sampling distribution is approximately normal in shape?

3. The time that people have to wait for an elevator in an office building has a uniform distribution over the interval from 0 to 1 minute. For this distribution, $\mu = 0.5$ and $\sigma = 0.289$
   (a) Let $\bar{x}$ be the average waiting time for a random sample of 16 waiting times. What are the mean and standard deviation of the sampling distribution of $\bar{x}$?

   (b) Answer Part (a) for a random sample of 50 waiting times. Draw a picture of the approximate sampling distribution of $\bar{x}$ when $n = 50$.

4. Suppose that a random sample of size 64 is to be selected from a population with mean 40 and standard deviation 5.
   (a) What are the mean and standard deviation of the $\bar{x}$ sampling distribution? Describe the shape of the $\bar{x}$ sampling distribution.
   (b) What is the approximate probability that $\bar{x}$ will be within 0.5 of the population mean μ?
   (c) What is the approximate probability that $\bar{x}$ will differ from μ by more than 0.7?

5. A sign in the elevator of a college library indicates a limit of 16 persons. In addition, there is a weight limit of 2,500 pounds. Assume that the average weight of students, faculty, and staff at this college is 150 pounds, that the standard deviation is 27 pounds, and that the distribution of weights of individuals on campus is approximately normal. A random sample of 16 persons from the campus will be selected.?
   (a) What are the mean and standard deviation of the $\bar{x}$ sampling distribution? Describe the shape of the $\bar{x}$ sampling distribution.
   (b) What average weights for a sample of 16 people will result in the total weight exceeding the weight limit of 2,500 pounds?
   (c) What is the probability that a random sample of 16 people will exceed the weight limit?

6. A manufacturing process is designed to produce bolts with a diameter of 0.5 inches. Once each day, a random sample of 36 bolts is selected and the bolt diameters are recorded. If the resulting sample mean is less than 0.49 or greater than 0.51, the process is shut down for adjustment. The standard deviation of bolt diameters is 0.02 inches. What is the probability that the manufacturing line will be shut down unnecessarily? (Hint: Find the probability of observing an $\bar{x}$ in the shutdown range when the actual process mean is 0.5 inches.)

7. What percentage of the time will a variable that has a t distribution with the specified degrees of freedom fall in the indicated region?
   (a) df=10, between -1.81 and 1.81
   (b) df=24, between -2.06 and 2.06
   (c) df=24, outside the interval from -2.80 to 2.80
   (d) df=10, to the left of -1.81

8. The formula used to compute a confidence interval for the mean of a normal population is

$$\bar{x} \pm (t\ criticalvalue)\frac{s}{\sqrt{n}}$$

   What is the appropriate t critical value for each of the following confidence levels and sample sizes?
   (a) 95% confidence, n=17
   (b) 99% confidence, n=24
   (c) 90% confidence, n=13

9. The two intervals (114.4, 115.6) and (114.1, 115.9) are confidence intervals for μ = mean resonance frequency (in hertz) for all tennis rackets of a certain type. The two intervals were computed using the same sample data.
   (a) What is the value of the sample mean resonance frequency?
   (b) The confidence level for one of these intervals is 90%, and for the other it is 99%. Which is which, and how can you tell?

10. A manufacturer of college textbooks is interested in estimating the strength of the bindings produced by a particular binding machine. Strength can be measured by recording the force required to pull the pages from the binding. If this force is measured in pounds, how many books should be tested to estimate the average force required to break the binding with a margin of error of 0.1 pound? Assume that σ is known to be 0.8 pound.

1.

n	$\mu_{\bar{x}}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
6	60	$3/\sqrt{6} = 1.22$
18	60	$3/\sqrt{18} = 0.71$
42	60	$3/\sqrt{42} = 0.46$
75	60	$3/\sqrt{75} = 0.35$
200	60	$3/\sqrt{200} = 0.21$
400	60	$3/\sqrt{400} = 0.15$

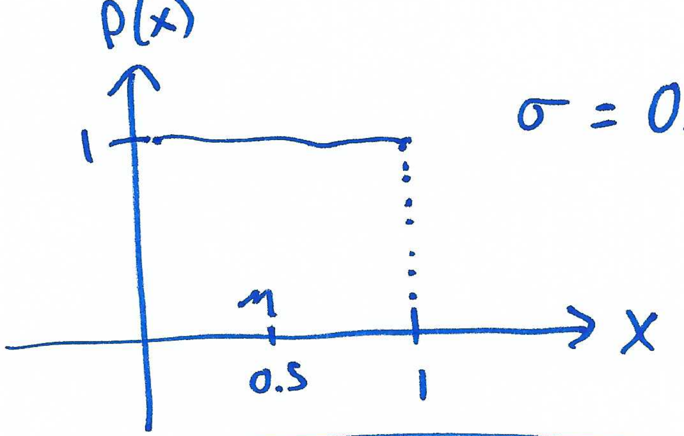
As the sample size increases, $\sigma_{\bar{x}}$ decreases.

As n increases, the sampling dist. of $\bar{x}$ (the graph) gets more narrowly centered around μ.

2. Any sample size greater than or equal to 30.

So, for $n = 42, 75, 200, 400$, the sampling distn. of $\bar{x}$ is approx. normal in shape.

③

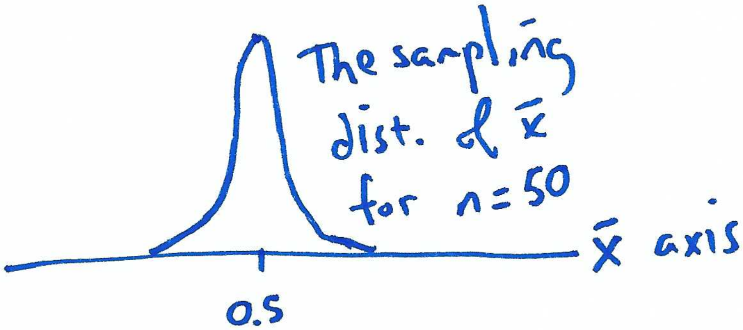


$\sigma = 0.289$

ⓐ $n = 16$; $\boxed{\mu_{\bar{x}} = 0.5}$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.289/\sqrt{16}$

$\boxed{\sigma_{\bar{x}} = 0.07225}$

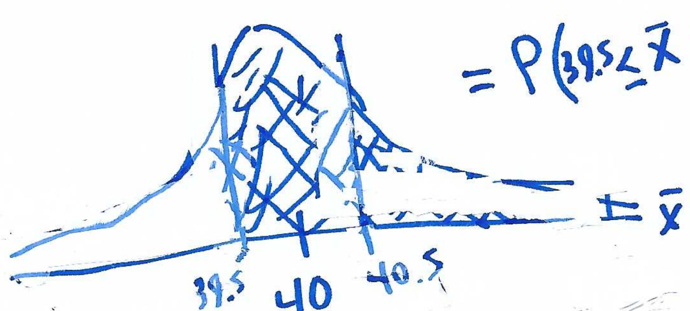
ⓑ $n = 50$; $\boxed{\mu_{\bar{x}} = 0.5}$ and $\boxed{\sigma_{\bar{x}} = 0.289/\sqrt{50} = 0.041}$



④ $n = 64$, $\mu = 40$, $\sigma = 5$

ⓐ $\mu_{\bar{x}} = 40$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 5/\sqrt{64} = 0.625$

ⓑ

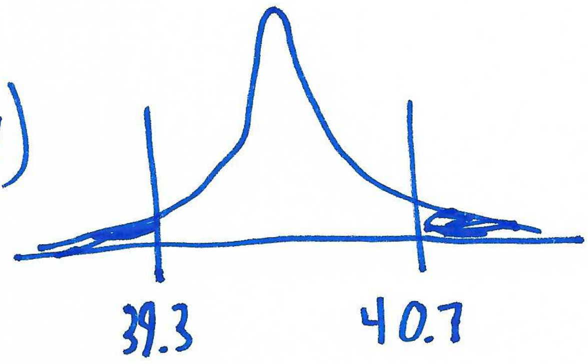


$= P(39.5 \leq \bar{x} \leq 40.5)$

$=$ normalcdf(39.5, 40.5, 40, .625)

$\approx \boxed{0.5763}$

(4c) $P(\bar{x} < 39.3 \text{ OR } \bar{x} > 40.7)$



$= 1 - P(39.3 < x < 40.7)$

$= 1 - \text{normalcdf}(39.3, 40.7, 40, 0.625)$

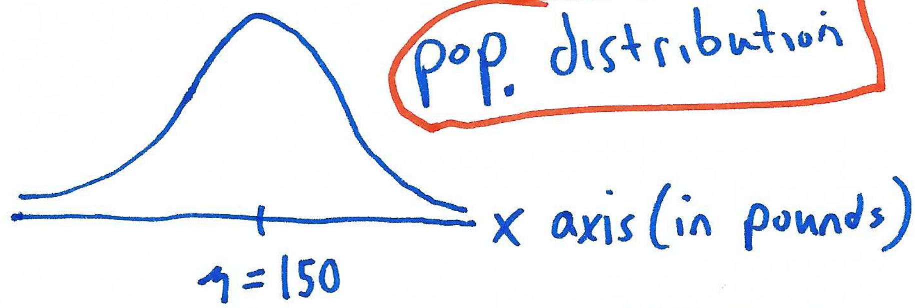
$= 1 - 0.7373 = \boxed{0.2627}$

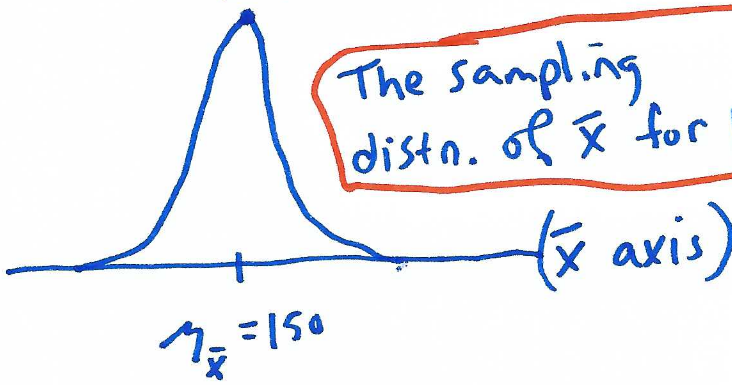
(5) $\mu = 150$ lbs., $\sigma = 27$ lbs., $n = 16$

(a) $\mu_{\bar{x}} = 150$ lbs, $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 27/\sqrt{16} = 27/4 = 6.75$ lbs.

Even though $n < 30$, since we are told that the population of weights, x, is normally distributed, we know (from the Central Limit Theorem) that the sampling distn. of $\bar{x}$ is approximately normally distributed.

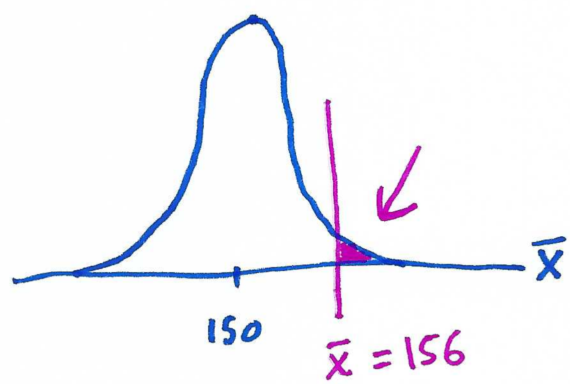
(5)





(b) $\dfrac{2{,}500 \text{ lbs}}{16} = \boxed{156.25}$ lbs.

(c) $P(\bar{x} > 156.25) =$



$= \text{normalCdf}(156.25, 10^9, 150, 6.75)$

$\approx \boxed{0.1772}$ (or 17.72%)

(6) $n = 36$, x = the continuous random variable representing bolt diameter, in inches.

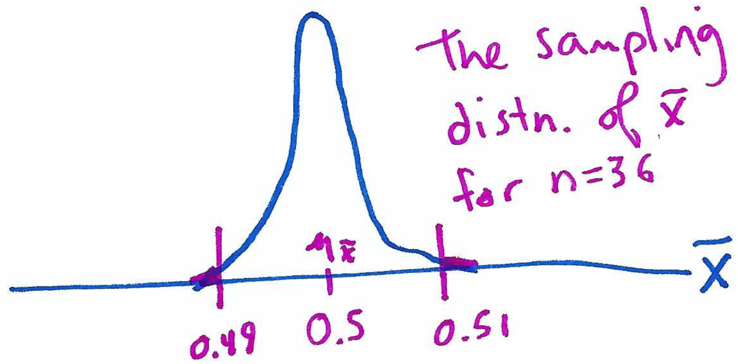
$\sigma = 0.02$ inches
$\mu = 0.5$ inches

Find $P(\bar{x} < 0.49 \text{ OR } \bar{x} > 0.51)$

Soln/

$P(\bar{x} < 0.49 \text{ OR } \bar{x} > 0.51)$

$=$



the sampling distn. of $\bar{x}$ for $n=36$

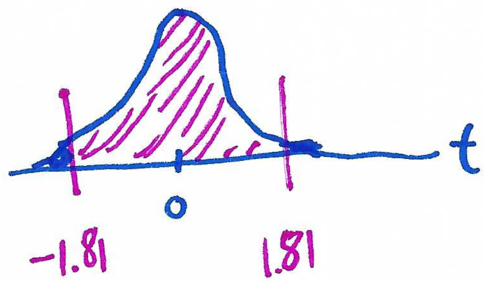
$= P(\bar{x} < 0.49) + P(\bar{x} > 0.51)$ | Since for mutually exclusive events A and B, $P(A \text{ or } B) = P(A) + P(B)$

$= 2 \cdot P(\bar{x} > 0.51)$ since the area in both tails are equal.

$= 2 \cdot \text{normalCDF}\left(0.51, 10^9, 0.5, 0.02/\sqrt{36}\right)$ $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

$\doteq 2 \cdot (0.0013) = \boxed{0.0027 \text{ (or } 0.27\%)}$

7 (a)

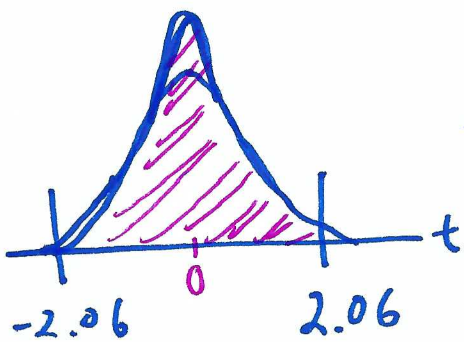


$\text{tcdf}(\text{lower}, \text{upper}, \text{df})$

$= \text{tcdf}(-1.81, 1.81, 10)$

$\approx 0.8996 \approx \boxed{90\%}$

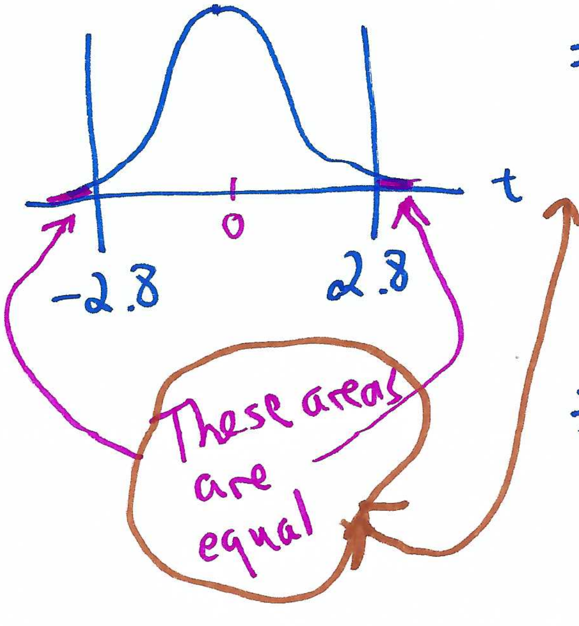
(b)



$\text{tcdf}(-2.06, 2.06, 24)$

$\approx 0.9496 \quad \boxed{95\%}$

(c)



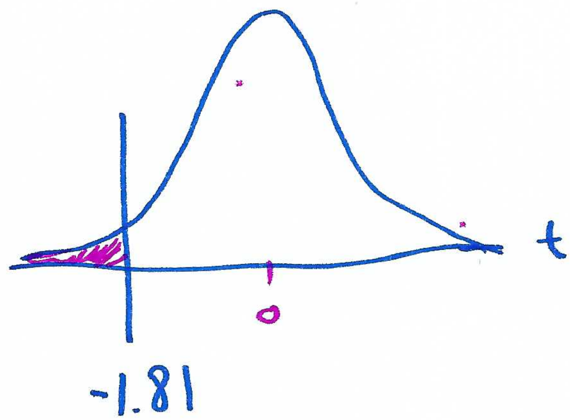
$= P(t < -2.8 \text{ or } t > 2.8)$

$= 2 \cdot P(t > 2.8)$

$= 2 \cdot \text{tcdf}(2.8, 10^9, 24)$

$\doteq 2 \cdot (0.005) = \boxed{0.01\ (1\%)}$

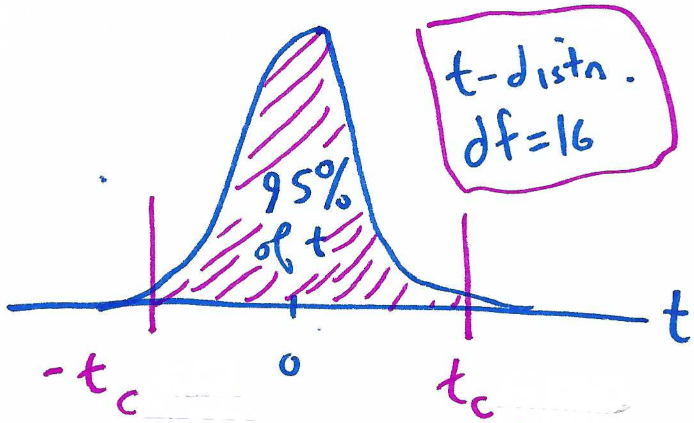
7d



$= tcdf(-10^{9}, -1.81, 10)$

$\approx \boxed{0.05 \text{ or } 5\%}$

8a Since $n=17$, $df = n-1 = 16$



$\boxed{\begin{array}{l}\text{t-distn.}\\ df=16\end{array}}$

$\boxed{t_{\text{critical value}} = t_{cv}}$

$$\left(\frac{\text{area left}}{\text{of } t_c}\right) = \frac{1+c}{2} = \frac{1+0.95}{2} = \frac{1.95}{2} = \boxed{0.975}$$

$$(df) = n-1 = \boxed{16}$$

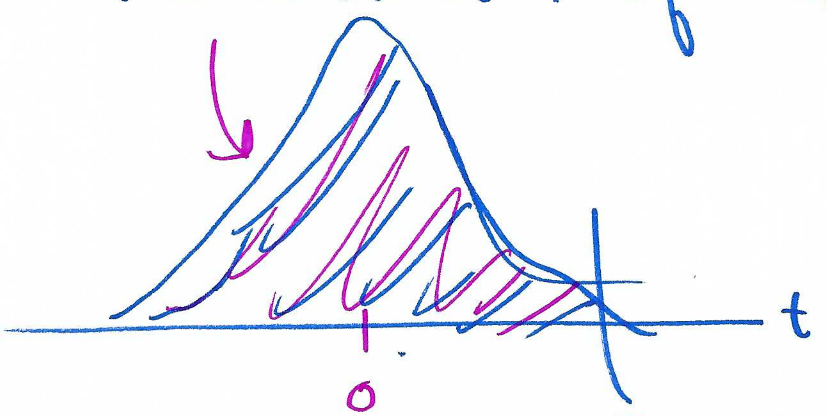
t-table or $InvT(\text{area}, df) = InvT(0.975, 16)$

2nd vars

$\doteq 2.1199 \doteq \boxed{2.12}$

8b) Since $n = 24$, $df = n - 1 = \boxed{23}$.

The area left of $t_{cv} = \frac{1+c}{2} = \frac{1+0.99}{2}$



$= 1.99/2 = \boxed{0.995}$

or $\boxed{99.5\%}$

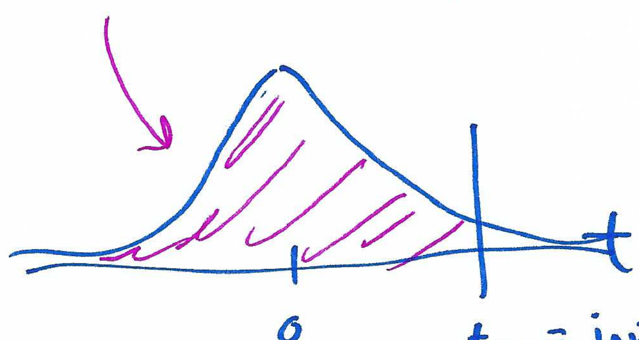
$t_{cv} = \text{invT}(\text{area}, df)$

$= \text{intT}(0.995, 23)$

$= \boxed{2.81}$ (rounded to hundredths)

8c) Since $n = 13$, $df = n - 1 = \boxed{12}$ and the area Left of $t_{cv} = \frac{1+c}{2} = \frac{1+0.90}{2} = \frac{1.9}{2} = \boxed{0.95}$

or $\boxed{95\%}$

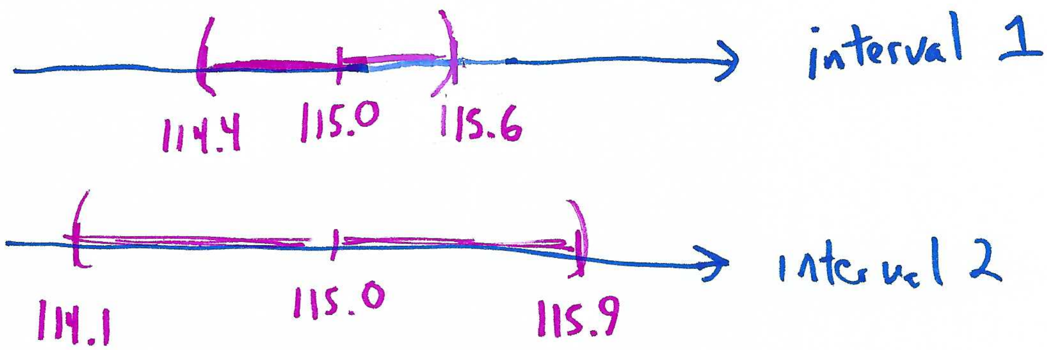


$t_{cv} = \text{invT}(0.95, 12) = \boxed{1.78}$

rounded to the hundredths

9a For interval 1, $\bar{x} = \dfrac{114.4 + 115.4}{2} = \boxed{115.0}$

and for interval 2, $\bar{x} = \dfrac{114.1 + 115.9}{2} = \boxed{115.0}$



9B

The formula for the margin of error is

$M = t_{cv} \cdot s/\sqrt{n}$. Since n is the same for both samples, t_{cv} is larger for interval 2, which, in turn makes M larger for interval 2.

The larger that the confidence level, c, is the larger (or wider) the interval. Therefore, the 2nd interval is the one with the 90% confidence level.

(10) X = the strength of the bindings produced by the machine (in pounds)

X is a continuous random variable.

$$n = \left(\frac{1.96\sigma}{M}\right)^2 = \left(\frac{1.96\cdot(0.8)}{0.1}\right)^2 = 245.86\ldots$$

$$\boxed{n = 246}$$

Always round this number <u>UP</u>!! to the <u>NEXT</u> whole number.

11. In a study of academic procrastination, the authors of the paper "Correlates and Consequences of Behavioral Procrastination" (Procrastination, Current Issues and New Directions [2000]) reported that for a sample of 411 undergraduate students at a mid-size public university, the mean time spent studying for the final exam in an introductory psychology course was 7.74 hours and the standard deviation of study times was 3.40 hours. Assume that this sample is representative of students taking introductory psychology at this university.

$n = 411$
$\bar{x} = 7.74$
$s = 3.4$

(a) Use the given information to estimate the mean time spent studying for the introductory psychology final exam.

(a) 7.74 hrs

(b) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

① The sample is representative of the population of study times

② The sample size is ≥ 30.

(c) Compute/find the value of the margin of error. (Use a 95% confidence level)

$$M = 1.96 \cdot s/\sqrt{n} = 1.96\left(3.4/\sqrt{411}\right) = 0.33 \text{ hrs}$$

(c) 0.33 hrs

(d) Interpret the meaning of the margin of error in the context of this problem.

It would be unusual for the estimate, $\bar{x}$, to differ from μ by more than 0.33 hrs, or 19.8 minutes.

(e) Construct a 95% confidence interval estimate of μ, the mean time spent studying for the introductory psychology final exam.

t- interval (7.4103, 8.0697)

(e) ______

(f) Communicate the Result: Interpret the confidence interval.

I am 95% confident that μ, the population average study time is between 7.41 hrs. and 8.07 hrs.

(g) Communicate the Result: Interpret the confidence *level.*

A method has been used to estimate the actual avg. study time, μ, that is successful in capturing μ 95% of the time.

Claim: $\mu > \$5000$

12. **Credit Card Balances** A credit card company claims that the mean credit card debt for individuals is greater than \$5000. You want to test this claim. You find that a random sample of 37 cardholders has a mean credit card balance of \$5122 and a standard deviation of \$625. At $\alpha = 0.05$, can you support the claim?

$n = 37$
$\bar{x} = 5122$
$s = 625$
$\alpha = 0.05$

(a) (2 points) Write the null and alternative hypotheses.

$H_0: \mu = \$5000$

$H_A: \mu > \$5000$

x = the continuous random variable representing credit card debt

(b) (2 points) What conditions should you check first before you conduct the hypothesis test?

① Is the sample random or representative of the pop?

② Is $n \geq 30$, or is the population approx. normal?

(c) (1 point) What formula should be used for the test statistic?

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

t-test on the Calculator

(d) (1 point) What number is the test statistic equal to?

$t = 1.187$

(e) (1 point) What p-value do you obtain? Round to the ten-thousandths.

P-val $= P(\bar{x} > 5122$, assuming $\mu_{\bar{x}} = 5000) = P(t > 1.187) = \boxed{0.1214}$

(f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.

Since the p-value $> \alpha$, fail to reject H_0.

(g) (2 points) Please write a conclusion sentence in the context of the problem.

There is not convincing sample evidence to suggest that the actual average credit card debt amount is higher than \$5000.

claim: $\mu < 5$

13. **Waste Generated** As part of your work for an environmental awareness group, you want to test a claim that the mean amount of waste generated by adults in the United States is less than 5 pounds per day. In a random sample of 19 adults in the United States, you find that the mean waste generated per person per day is 4.43 pounds with a standard deviation of 1.21 pounds. At $\alpha = 0.01$ can you support the claim? Assume the population is normally distributed.

$n = 19$

$\bar{x} = 4.43$

$s = 1.21$

(a) (2 points) Write the null and alternative hypotheses.

$$\boxed{\begin{array}{l} H_0: \mu = 5 \\ H_A: \mu < 5 \end{array}}$$

(b) (2 points) What conditions should you check first before you conduct the hypothesis test?

① The sample is random, or representative of the pop.

② Either $n \geq 30$, or the pop is normally distributed.

(c) (1 point) What formula should be used for the test statistic?

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

(d) (1 point) What number is the test statistic equal to? $\boxed{t = -2.05}$

(e) (1 point) What p-value do you obtain? Round to the ten-thousandths.

P-val $= P(\bar{x} < 4.43$ assuming $\mu = 5) = P(t < -2.05) = \boxed{0.0274}$

(f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.

Since the p-val $> \alpha$, fail to reject H_0

(g) (2 points) Please write a conclusion sentence in the context of the problem.

There is not convincing sample evidence to suggest that the actual average amount of waste generated each day (by a person in the U.S.) is less than 5 lbs. per day.

Class sizes					
35	28	29	33	32	40
26	25	29	28	30	36
33	29	27	30	28	25

claim: $\mu < 32$

14. **Faculty Classroom Hours** Class Size You receive a brochure from a large university. The brochure indicates that the mean class size for full-time faculty is fewer than 32 students. You want to test this claim. You randomly select 18 classes taught by full-time faculty and determine the class size of each. The results are shown in the table at the left. At $\alpha = 0.05$, can you support the university's claim? Assume the population is normally distributed.

(a) (2 points) Write the null and alternative hypotheses.

$$\boxed{\begin{array}{l} H_0: \mu = 32 \\ H_A: \mu < 32 \end{array}}$$

(b) (2 points) What conditions should you check first before you conduct the hypothesis test?

① The sample is random, or representative of the pop.

② Either $n \geq 30$, or the pop. is approx. normally distr.

(c) (1 point) What formula should be used for the test statistic?

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

(d) (1 point) What number is the test statistic equal to? $\boxed{t = -1.94}$

(e) (1 point) What p-value do you obtain? Round to the ten-thousandths.

P-val $= P(\bar{x} < 30.1\overline{6}$, assuming $\mu = 32) = P(t < -1.94) = \boxed{0.0344}$

(f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.

Since the p-val $\leq \alpha$, reject H_0

(g) (2 points) Please write a conclusion sentence in the context of the problem.

There is convincing sample evidence that suggests that the avg. class size is fewer than 32 students.